Let $\dot{\theta}$ = the angular motion of the instantaneous axis on the plane; this is the angular motion about the line (n) which is normal to the inclined plane and passes through the vertex of the cone. A plane perpendicular to the inclined plane includes this normal, the instantaneous axis (a_i), the axis of the cone, and a perpendicular (p) to the axis of the cone. The angle between n and p is α and between a_i and p is $\pi/2 + \alpha$. The components of $\dot{\theta}$ and ω about p are $\dot{\theta}\cos\alpha$ and $\omega\cos(\pi/2+\alpha) = -\omega\sin\alpha$, but these denote the same motion; then

$$\dot{\theta}\cos\alpha = -\omega\sin\alpha \qquad \text{or} \qquad \omega = -\dot{\theta}\cot\alpha. \tag{6}$$

Now the moment of the external forces about the instantaneous axis

$$= L\cos\lambda + M\cos\mu + N\cos\nu; \tag{7}$$

also

$$= \tfrac{3}{4}mgh\sin\alpha\sin\beta\sin\theta \tag{8}$$

β being the inclination of the plane.

Substituting (4) in (3) and differentiating, we have

$$\dot{\omega}_1 = \sin\alpha(\cos\varphi\cdot\dot{\varphi}^2 + \sin\varphi\cdot\ddot{\varphi}),\quad \dot{\omega}_2 = \sin\alpha(-\sin\varphi\cdot\dot{\varphi}^2 + \cos\varphi\cdot\ddot{\varphi}),\quad \text{and}\quad \dot{\omega}_3 = \cos\alpha\cdot\ddot{\varphi}, \tag{9}$$

Now put the values given by (3), (5), (9) in (1), and we have the values of L, M, N; and using these, with (2), (7) and (8), we have the single equation of motion

$$\frac{3m}{20}(6+\tan^2\alpha)h^2\sin^2\alpha\cdot\ddot{\varphi} = \frac{3m}{4}gh\sin\alpha\sin\beta\sin\theta. \tag{10}$$

Now from (4) and (6),

$$\ddot{\theta} = -\frac{g}{h}\frac{5\sin\beta\cos\alpha}{1+5\cos^2\alpha}\sin\theta. \tag{11}$$

If θ be so small that θ may displace $\sin\theta$, we have the required time of oscillation

$$T = 2\pi\sqrt{\frac{1+5\cos^2\alpha}{5\sin\beta\cos\alpha}\cdot\frac{h}{g}}.$$

Also solved by F. L. Wilmer, and discussed by C. H. Eckart.

2820 [1920, 134]. Proposed by C. B. HALDEMAN, Ross, Ohio.

Given one angle and the radii of the inscribed and circumscribed circles, to construct the triangle geometrically.

Solution by A. V. Richardson, Bishop's College, Lennoxville, Quebec.

With the usual notation, A, R, r are given. Draw a circle of radius R, and let KB be any diameter. (The reader is requested to draw the figure). Make the angle, $BKC = A$. Draw XY parallel to BC and at a distance r from it on the same side as K. Bisect the arc BC at V, and let the circle, center V, radius VC, cut XY at I and I'.

These points will be the incenters for the two (symmetrical) solutions. Let VI meet the circle again in A. Then ABC is the required triangle. To show this, it is only necessary to prove that the straight line CI bisects the angle C. Since $VI = VC$, $\angle VIC = \angle VCI$. But $\angle VIC = (A/2) + \angle ICA$ and $\angle VCI = (A/2) + \angle BCI$. Hence $\angle ICA = \angle BCI$ and I is the intersection of two bisectors of the angles of the triangle ABC.

Also solved by L. C. Mathewson, H. L. Olson, Arthur Pelletier, J. B. Reynolds, Joseph Rosenbaum, C. N. Schmall, and the Proposer.

2829 [1920, 226]. Proposed by E. S. PALMER, New Haven, Conn.

Given a set of arbitrary pairs of positive integers (a_p, b_p), $(p = 1, 2, \cdots, n)$: (a) Is it always possible to find a set of positive integers k_p, $(p = 1, 2, \cdots, n)$ such that

$$k_pa_p + k_pb_p > \sum_{r=1}^{r=n} k_ra_r, \qquad (p = 1, 2, 3, \cdots, n).$$

(b) If or when possible, show how to find k_p.

Solution by Albert A. Bennett, University of Texas.

The system is homogeneous, so that if one solution exists, an infinite number exist. For convenience write c_p for $a_p + b_p$ and write in place of the inequalities the following equalities:

$$k_p c_p = \sum_{r=1}^{n} k_r a_r + e_p, \qquad e_p > 0, \qquad (p = 1, 2, 3, \cdots, n).$$

Solving for k_p, one has, provided that the denominators do not vanish,

$$k_p = \frac{e_p}{c_p} + \frac{1}{c_p} \frac{\sum_{r=1}^{n} \frac{e_r a_r}{c_r}}{1 - \sum_{r=1}^{n} \frac{a_r}{c_r}},$$

as may be verified at once by substitution.

Three cases occur.

1. $1 - \sum_{r=1}^{n} \frac{a_r}{c_r} > 0$. In this case, solutions are obtained readily by inserting arbitrary e's restricted only so that the k's shall be integers.

2. $1 - \sum_{r=1}^{n} \frac{a_r}{c_r} = 0$. There are no solutions in this case.

3. $1 - \sum_{r=1}^{n} \frac{a_r}{c_r} < 0$. In this case, also, there are no solutions. For if there were solutions, these would be in the form given above, and the expression, $\sum_{p=1}^{n} k_p a_p$, obtained by multiplying each solution, k_p by a_p and adding together would be

$$\sum_{p=1}^{n} k_p a_p = \sum_{p=1}^{n} \frac{e_p a_p}{c_p} + \sum_{p=1}^{n} \frac{a_p}{c_p} \frac{\sum_{r=1}^{n} \frac{e_r a_r}{c_r}}{1 - \sum_{r=1}^{n} \frac{a_r}{c_r}},$$

$$= \sum_{r=1}^{n} \frac{e_r a_r}{c_r} \frac{1}{1 - \sum_{r=1}^{n} \frac{a_r}{c_r}}.$$

Since the right-hand member would be negative under the hypothesis of this case, there could be no set of positive members, k_p, $p = 1, 2, \cdots, n$, of the form required for a solution. (The writer is indebted to Prof. H. P. Manning for the treatment here given of this third case.)

2833 [1920, 227]. Proposed by W. H. ECHOLS, University of Virginia.

In *Engineering*, London, September 28, 1917, appeared the following equations concerning the stability of ships; they are employed by the naval constructors in the Norfolk (Virginia) Navy Yard, and they are of importance:

$$\tan^3 \theta_0 + \frac{2m}{\rho} \tan \theta_0 - \frac{2x_0}{\rho} = 0, \qquad \tan^3 (\theta_0 + \theta_1) + \frac{2m}{\rho} \tan (\theta_0 + \theta_1) - \frac{2x_0}{\rho} - \frac{2x}{\rho} = 0,$$

$$\tan^3 (\theta_0 - \theta_2) + \frac{2m}{\rho} \tan (\theta_0 - \theta_2) - \frac{2x_0}{\rho} + \frac{2x}{\rho} = 0.$$

The required unknowns are θ_0, x_0 and m. The constants have values as follows: θ_1 and θ_2 are positive angles ranging from 15′ to 10°, x is positive and less than 5, and ρ is positive with a considerable range of values. A rapid solution involving small labor is desired, determining x_0 within the same limits as given for x.

Solution by the Proposer.

It may be of interest to note that in the practical application of the problem, ρ is the metacentric radius when the vessel is upright, x is the shift of the center of gravity of the ship when a weight w is placed on the side at a distance d from the center line and is equal to wd/W, where W is the weight of displaced water. In a typical ship of 4000 tons, w equal to 2.5 tons, $\rho = 16$ ft.,